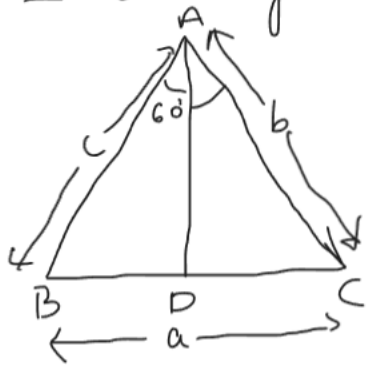
In a triangle ABC, AD is the angle bisector of $\angle BAC$ & $\angle BAD = 60^\circ$. What is length of AD?



AD is angle bisector

so $\angle BAD = \angle DAC = 60^\circ$

also $BD = DC$

$BD + DC = a$

$BD + BD = a$

$\boxed{BD = \frac{a}{2}}$

Let $AD = h$

Area of $\triangle^{le} ABC = \frac{1}{2} bc \sin 120^\circ = \frac{\sqrt{3}}{4} bc$

Area of $\triangle^{le} BAD = \frac{1}{2} ch \sin 60^\circ = \frac{\sqrt{3}}{4} ch$

" " $CAD = \frac{1}{2} bh \sin 60^\circ = \frac{\sqrt{3}}{4} bh$

Now Area of $\triangle^{le} ABC$ = Area of $\triangle^{le} BAD$ + Area of $\triangle^{le} CAD$

$$\frac{\sqrt{3}}{4} bc = \frac{\sqrt{3}}{4} ch + \frac{\sqrt{3}}{4} bh \Rightarrow bc = ch + bh$$

$$bc = h(b+c)$$

AD $\Rightarrow \boxed{h = \frac{bc}{b+c}}$